#1)

a)

Compute ∇f for

$$\nabla f = f_x \hat{\imath} + f_y \hat{\jmath},$$
$$= (60x - 60xy^2)\hat{\imath} + (60y - 60x^2y)\hat{\jmath}.$$

Set $\nabla f = 0$ to find critical points for

$$\begin{cases} x - xy^2 = 0 \\ y - x^2y = 0 \end{cases} \Rightarrow \begin{cases} x(1-y^2) = 0 \\ y(1-x^2) = 0 \end{cases}$$

So the critical points are $(0,0)$ & $(\pm 1, \pm 1)$.

The minimum (bottom) is at $(0,0,8000)$.

Identifying the point

Justifying

$$H = 60^2 \begin{vmatrix} 1-y^2 & -2xy \\ -2xy & 1-x^2 \end{vmatrix} = [(1-y^2)(1-x^2) - 4x^2y^2]60^2$$

So, $H|_{(0,0)} = 60^2 > 0$ & $f_{xx} = 60 > 0$ & $(0,0,8000)$ is a minumum.

b)

Identifying $(\pm 1, \pm 1)$

Justifying is also 4 pt:

$$H|_{(\pm 1, \pm 1)} = -4(60^2) < 0,$$

so they are saddle points.

c) Since $f(\pm 1, \pm 1) = 8030$ & $f(0,0) = 8000$, max -

imum depth is $8030 - 8000 = 30$.
If they wrote 8030, we marked off

#2 a) $\nabla f \cdot \hat{i} = 5 \longrightarrow f_x(0,0) = 5$

$\nabla f \cdot \hat{j} = 12 \longrightarrow f_y(0,0) = 12$

Then $\nabla f = 5\hat{i} + 12\hat{j} \longrightarrow |\nabla f| = 13$

$$\frac{\nabla f}{|\nabla f|} = \frac{5}{13}\hat{i} + \frac{12}{13}\hat{j}$$

Max: $50 + 13 = 63$ @ $\left(\frac{5}{13}, \frac{12}{13}\right)$

Min: $50 - 13 = 47$ @ $\left(\frac{-5}{13}, \frac{-12}{13}\right)$

#3

Objective $\qquad\qquad$ Constraint

$f = x + y^2 \qquad\qquad g = y - (x-1)^2 = 0$

$$\nabla f = \lambda \nabla g$$

$$\hat{\imath} + 2y\hat{\jmath} = \lambda\left(-2(x-1)\hat{\imath} + \hat{\jmath}\right)$$

Need to solve

① $1 = -2\lambda(x-1)$

② $2y = \lambda$

③ $y = (x-1)^2$

from ② $y = \lambda/2$ and from ① $x - 1 = -1/2\lambda$

put both into ③ to get $\lambda/2 = (-1/2\lambda)^2$

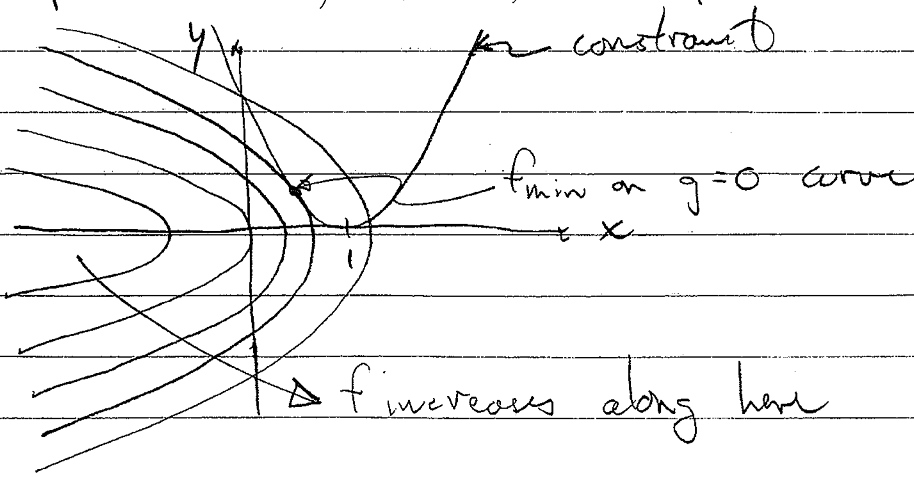
or $\lambda^3 = 1/2$

so $\lambda = 1/2^{1/3}$

Thus $y = 1/2 \cdot 2^{1/3} = 1/2^{4/3}$

$x = 1 - 1/2^{2/3}$

Thus $f|_p = \left(1 - \frac{1}{2^{2/3}}\right) + \left(1/2^{8/3}\right)$



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a) $f_x = 2x + 4x^3y^4$ $\qquad f_y = 2y + 4x^4y^3$

$f_{xx} = 2 + 12x^2y^4 \qquad f_{xy} = 16x^3y^3 \qquad f_{yy} = 2 + 12x^4y^2$

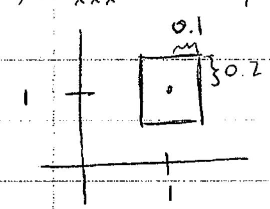
$$f(x,y) \approx 3 + 6(x-1) + 6(y-1) + \frac{1}{2}\left[14(x-1)^2 + 2(16)(x-1)(y-1) + 14(y-1)^2\right]$$

$$\approx \boxed{7x^2 + 16xy + 7y^2 - 24x - 24y + 21}$$

b) $$f(1.1, 1.2) \approx 3 + 6(0.1) + 6(0.2) + \frac{1}{2}\left[14(0.01) + 32(0.02) + 14(0.04)\right]$$

$$\approx \boxed{5.47}$$

c) $f_{xxx} = 24xy^4 \qquad f_{xxy} = 48x^2y^3 \qquad f_{xyy} = 48x^3y^2 \qquad f_{yyy} = 24x^4y$



$$M_{ax} = f_{xxy}(1.1, 1.2) = 100.36224$$

$$|E(x,y)| \le \frac{M}{2}(0.1 + 0.2)^2 = \boxed{0.45163}$$

Problem 5

a. $\frac{dx}{x} = .01 \qquad \frac{dy}{y} = .02 \qquad \frac{dz}{z} = .03$

(a) $$\frac{df}{f} = \frac{y^2\,dx}{xy^2} + \frac{2yx\,dy}{xy^2} = \frac{dx}{x} + \frac{2dy}{y} = .01 + .04 = .05$$
$$= \boxed{5\%}$$

(b) $$\frac{df}{f} = \frac{3zy^2\,dy}{zy^3} + \frac{y^3\,dz}{zy^3} = \frac{3dy}{y} + \frac{dz}{z} = .06 + .03$$
$$= .09$$
$$= \boxed{9\%}$$

(c) $$\frac{df}{f} = \frac{\frac{3x^2}{z^2}\,dx}{x^3/z^2} + \frac{-\frac{2x^3}{z^3}\,dz}{x^3/z^2} = \frac{3dx}{x} - \frac{2dz}{z}$$
$$= 0.03 - 0.06 = -.03$$
$$= \boxed{-3\%}$$

b. You should pick equation (c) and divide the error of z by 2.

$$\frac{df}{f} = \frac{3dx}{x} - \left(\frac{1}{2}\right)\frac{dz}{z} = 0.03 - 0.03 = 0$$
$$= \boxed{0\%}$$