1) a) $h = xe^{-(x^2+y^2)}$ so $h_x = e^{-(x^2+y^2)} - 2x^2e^{-(x^2+y^2)} = 0$

and $h_y = -2yxe^{-(x^2+y^2)} = 0$

gives $y=0$ and $x = \pm 1/\sqrt{2}$

Crit. pts are $(1/\sqrt{2}, 0)$ + $(-1/\sqrt{2}, 0)$

b) $h_{xx} = e^{-(x^2+y^2)}(4x^3 - 6x)$

$h_{xy} = e^{-(x^2+y^2)}(4yx^2 - 2y)$

$h_{yy} = e^{-(x^2+y^2)}(4y^2x - 2x)$

At $(-1/\sqrt{2}, 0)$ $D = h_{xx}h_{yy} - h_{xy}^2 > 0$ and $h_{xx} > 0$

so $(-1/\sqrt{2}, 0)$ is a rel. min.

At $(1/\sqrt{2}, 0)$ $D > 0$ and $h_{xx} < 0$

so $(1/\sqrt{2}, 0)$ is a rel. max.

c) $h(-1/\sqrt{2}, 0) = -1/\sqrt{2e}$ and $h(1/\sqrt{2}, 0) = 1/\sqrt{2e}$

d) $\nabla h\Big|_{(0,0)} = h_x\Big|_{(0,0)}\hat{i} + h_y\Big|_{(0,0)}\hat{j} = \hat{i}$ so move in $\hat{i}$ direction.

$\frac{df}{ds} = |\nabla h|\Big|_{(0,0)} = 1$ if you move in $\hat{i}$ direction.

e) Move orthogonal to ∇h, ie in the $\hat{j}$ or $-\hat{j}$ direction.

//

2) a) $P = (\text{total sales}) - (\text{expenses})$

$f = xyz^2 - 10000(x+y+z)$ objective

b) $g = x+y+z = 8000$ constraint (4)

c) $\nabla f = \lambda \nabla g$ gives

$$yz^2 - 10000 = \lambda \quad (1)$$
$$xz^2 - 10000 = \lambda \quad (2)$$
$$2xyz - 10000 = \lambda \quad (3)$$

from (1) and (2) $yz^2 - 10000 = xz^2 - 10000$

$$z^2(y-x) = 0$$

$\underbrace{z=0 \text{ (lose money)}}_{\text{don't want}}$ $\qquad y = x$ (5)

from (3) and (2) $xz^2 - 10000 = 2x^2z - 10000$

$$xz(z-2x) = 0$$

$\underbrace{x=0 \qquad z=0}_{\text{lose money on both so don't want}}$ $\qquad 2x = z$ (6)

Put (5) and (6) into constraint (4)

$$x+y+z = x+x+2x = 4x = 8000$$
$$x = 2000$$

$\therefore\ x = y = 2000 \qquad z = 4000$

Check: $P\big|_{2000,2000,4000} = 64\times10^{12} - 10^4(8\times10^3)$, but

at another pt on constraint, like $(4000, 2000, 2000)$,

$P\big|_{4000,2000,2000} = 32\times10^{12} - 10^4(8\times10^3)$ which is smaller.

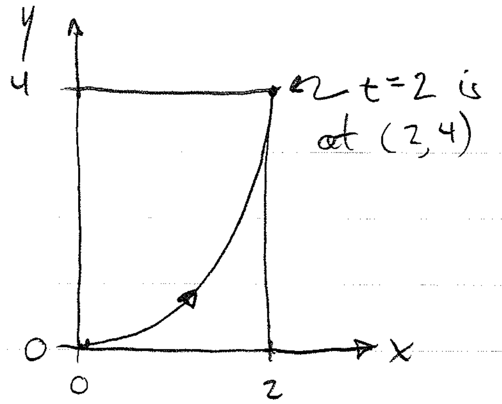
Thus we found a max.

3

$\underline{r} = t\,\hat{i} + t^2\,\hat{j}$ $\qquad T = xy$

$\underline{V} = \hat{i} + 2t\,\hat{j}$ $\qquad \nabla T = y\hat{i} + x\hat{j}$

$|\underline{V}| = \sqrt{1+4t^2}$



a) $$\left.\frac{dT}{dt}\right|_{path} = \nabla T \circ \underline{V}$$

$$= (y\hat{i} + x\hat{j}) \circ (\hat{i} + 2t\hat{j})$$

$$= (t^2\hat{i} + t\hat{j}) \circ (\hat{i} + 2t\hat{j}) = 3t^2 \quad (\text{on path } \underline{r}(t))$$

Max. of $\frac{dT}{dt} = 3t^2$ on path $(0 \le t \le 2)$ occurs when $t=2$ in which case $dT/dt = 12$.

b) $$\frac{dT}{ds} = \frac{dT}{dt}\cdot\frac{dt}{ds} = \frac{dT}{dt}\,\frac{1}{|\underline{V}|} = \frac{3t^2}{\sqrt{1+4t^2}}$$

Max. of $\frac{dT}{ds} = \frac{3t^2}{\sqrt{1+4t^2}}$ for $0 \le t \le 2$ occurs when $t=2$ in which case $dT/ds = 12/\sqrt{17}$.

c) Since $\frac{dT}{ds} = \nabla T \circ \hat{u}$ and ∇T depends only on field $T=xy$ and $\hat{u}$ depends only on shape of path then dT/ds is indep. of speed.

However $\frac{dT}{dt} = \nabla T \circ \underline{V}$ and $\underline{V}$ depends on speed, so dT/dt will change.

In fact $\frac{dT}{dt} = \frac{dT}{ds}\cdot\frac{ds}{dt} = \nabla T \circ \hat{u} \cdot |\underline{V}|$ indicates that dT/dt will double if speed doubles.

4)

$f = x^3y^3 + x^2y^2 = 2$

$f_x = 3x^2y^3 + 2xy^2 = 5 \qquad f_y = 3x^3y^2 + 2x^2y = 5$

$f_{xx} = 6xy^3 + 2y^2 = 8 \qquad f_{yy} = 6x^3y + 2x^3 = 8$

$f_{xy} = 9x^2y^2 + 4xy = 13$

values at (1,1)

a) $f(x,y) \approx 2 + [5(x-1) + 5(y-1)] + \frac{1}{2}[8(x-1)^2 + 26(x-1)(y-1) + 8(y-1)^2]$ (*)

$= 4x^2 + 4y^2 + 13xy - 16x - 16y + 13$

b) $f(x,y) = (*)\Big|_{(1.1,\,1.1)} = 2 + 5[0.1 + 0.1] + \frac{1}{2}[8(0.1)^2 + 26(0.1)^2 + 8(0.1)^2]$

$= 3.21$

c) $f_{xxx} = 6y^3$

$f_{xxy} = 18xy^2 + 4y$

$f_{xyy} = 18x^2y + 4x$

$f_{yyy} = 6x^3$

find max. magnitude of these for $|x-1| \le 0.1$ and $|y-1| \le 0.1$

By inspection this will be

$M = f_{xxy}\Big|_{(1.1,\,1.1)} = 18(1.1)^3 + 4(1.1)$

then

$|\text{error}| \le \frac{M}{3!}\left[|x-1| + |y-1|\right]^3 = \frac{M}{6}(0.2)^3$